

Activity I: How Does a Radio Work? lesson overview

Materials: handouts, antennae and TV / radio (for demonstration purposes), graphing calculator

Time for Set-Up: none

Time for Lesson: Part A: 30 minutes, Part B: 45 minutes, Part C: 20 minutes, Part D: 2 hours

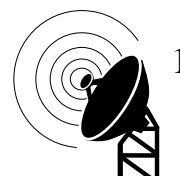
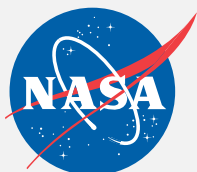
Student Prerequisites: Part A: knowledge of parallel and perp line equations, circle equations, Part B: basic probability, Part C: algebra with order of operations, exponents, reciprocals, radicals, Part D: ability to use graphing calculators to enter equations and produce graphs

Icons for recommended subject areas where activities could be used:
Part A: A1, GEO, PHYS; Part B: STAT; Part C: A1, PHYS, A2; Part D: TRIG, PHYS

Objectives / Link to Standards Matrix:

- * Students will write equations for lines, using knowledge about parallel and perpendicular slope relationships and given points.
- * Students solve systems of linear equations and linear/circle combinations.
- * Students will determine the equations of circles.
- * Students will use a data set to create a scatter plot and determine the linear relationship between two variables (line of regression).
- * Students will determine the probability of events occurring given a random or Poisson distribution.
- * Students will determine the probability of events occurring given a non-random distribution.
- * Students will weigh several probability scenarios, determining which one is more realistic.
- * Students will solve for an unknown variable in a complex equation with square roots and a constant (π), by a process including evaluation.
- * Students will use inductive reasoning to establish relationships between equation characteristics and the associated graph.
- * Students will learn the basic graphical elements of a trigonometric function and its associated equation.
- * Students will be able to estimate the equation that corresponds to a graph, for a sine-related trigonometric function.
- * Students will learn how radio wave electric and magnetic field orientation relative to antennae, as well as capacitance and inductance, influence the quality and frequency of signal received.

Student Assessments: calculations, scatterplot and line graphs, prose, graphs.





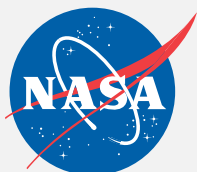
Communication

teacher guide

Activity I: How Does a Radio Work? introduction

Introduction

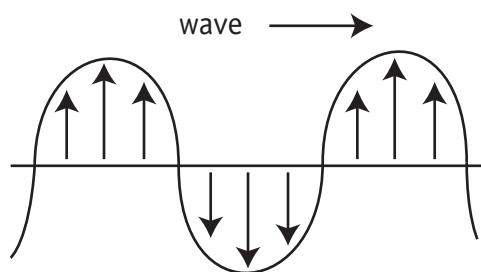
The Communication section investigates how radios work. Specifically, how tuning a radio to receive a new frequency is related to the position of the antennae, capacitor, and inductor. The activity **How Does a Radio Work?** is broken up into 4 parts. **Part A. The Importance of Parallel and Perpendicular Orientation In Reception** employs knowledge about perpendicular and parallel planes and lines, as well as linear and circular equations, to teach how antennae position is important. A small statistics section, **Part B. Probability and Reception**, looks at the probability of getting ideal reception per unit time when gradually turning a circular TV or radio antennae. In **Part C. Understanding the Relationships Between Frequency, Capacitance, and Inductance**, students use algebra with radicals and pi to understand that the positions of the capacitor and inductor influence the frequency that a radio will receive and transmit. In **Part D. Understanding Components of Waves**, the main characteristics of a wave are covered, and students are urged to use their graphing calculators to discover how elements of wave equations are related to structural elements of waves.



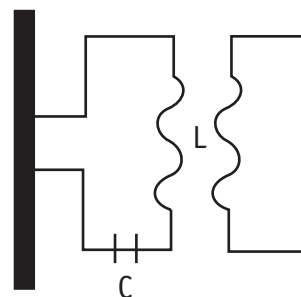
Activity I: How Does a Radio Work? introduction

When sound is passed from one location (where it originates) to another, it is sent in the form of waves. These waves reach receiving antennae and interact with the electric charges in the antennae wires. The sound waves have electric and magnetic fields that can interact with the wires.

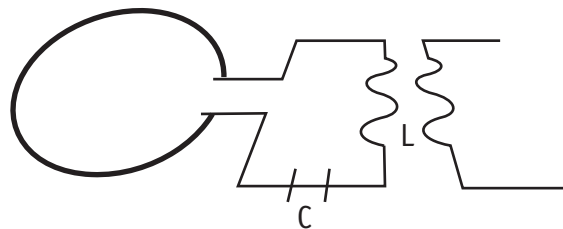
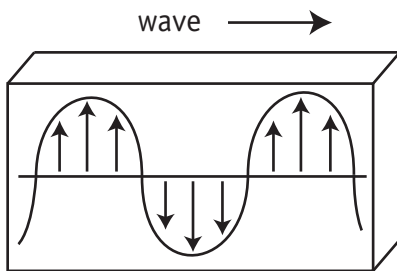
In order for the electric field to interact with the wires, the wires must be parallel to the field. The electric field acts on the electrons in the receiving wire, and by doing so an Alternating Current is generated.



Electric Field direction is parallel to receiving wires



In order for the magnetic field to interact with the receiving wires, the receiving wires must be in a loop. For best reception, the plane of the loop is oriented perpendicular to the plane of the magnetic field. This way the loop cuts through the magnetic field. As the magnetic field comes in contact with the loop, magnetic flux is altered, inducing a voltage and current in the loop, in accordance with Faraday's law.



Magnetic Field is parallel to normal of antenna loop's plane

In order to selectively limit the frequency (f_0) of waves received by antennae, the variable capacitor (C) and inductor (L) can be altered. In radios, generally the inductor is fixed, and turning a knob changes the capacitor. Dozens of radio frequencies come in contact with a single antenna, but only the radio frequency ("resonance frequency") matching the frequency to which the antenna is set will be result in good reception because this frequency will produce the most current in the antenna.



Activity I: How Does a Radio Work?

Part A - The importance of parallel and perpendicular orientation in reception

Let us assume that no reception will take place if the proper parallel or perpendicular orientation does not occur.

1. If the plane of the electric field moves along a line of the equation $y = 2x + 3$, what do we know about the slope of the receiving wires?

Answer: Slope is 2 because the two lines (planes) must be parallel (meaning they have the same slope).

2. If the receiving wires contain the point (3 , 7), what is the equation for the line that contains the receiving wires?

Answer: $y = 2x + 1$

Work: $y = 2x + b$

$$7 = 2(3) + b$$

$$b = 1$$

3. If the magnetic field moves along a plane with a slope of -2, what do we know about the slope of the receiving loop's diameter?

Answer: It is 0.5 because the two slopes must multiply to make -1 in order to be perpendicular.

4. If the receiving loop's diameter contains the point (5 ,7), what is the equation for a line that contains this diameter?

Answer: $y = -2x + 17$

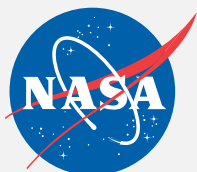
Work: $y = -2x + b$

$$7 = -2(5) + b$$

$$b = 17$$

5. If (5 ,7) is the center of the circle with a radius of 10, what is the equation for the loop (assume it is a full circle)?

Answer: $100 = (x - 5)^2 + (y - 7)^2$



Activity I: How Does a Radio Work?

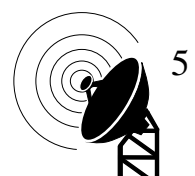
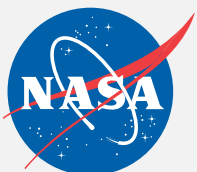
6. At what two points do the loop and magnetic field cross if it passes through the center of the circular loop?

The magnetic field's line: $y = 0.5x + b$
 $7 = 0.5(5) + b$
 $b = 4.5$
 $y = 0.5x + 4.5$

Substitution in equation of loop:
 $100 = (x - 5)^2 + (0.5x + 4.5 - 7)^2$
 $100 = x^2 - 10x + 25 + 0.25x^2 - 2.5x + 6.25$
 $0 = x^2 - 10x - 55$

Quadratic equation to solve, then substitute x to solve for y:

$(5 + 4(5)^{1/2}, 7 - 2(5)^{1/2})$ and $(5 - 4(5)^{1/2}, 7 + 2(5)^{1/2})$
 $(5 + 4(5)^{1/2}, 7 - 2(5)^{1/2})$



Activity I: How Does a Radio Work?

Now let us assume that the receiving loop does not have to be perpendicular to the linear path that the magnetic field travels along. (You probably recall that when adjusting an antennae on a TV, the loop can be tuned away from a signal, yet some signal is still received!) The amount of signal received, however, is still directly related to the angle between the magnetic field's plane of travel and the loop's plane. Although the amount of signal will vary quite a bit, for simplicity's sake, let us say that at the following angles, you receive the following percent of signal. See the chart below.

Note: The following values are not true, but they are realistic. Obtain data yourself by doing an experiment!

Orientation and Signal

Angle Between Two Planes (degrees)	Percent of Signal Received
90	100
85	94.44
70	77.78
60	66.67
45	50
30	33.33
25	27.78
18	20
10	11.11
0	0

7. Draw this relationship using a line graph. Let angle be x , and signal be y .

8. What is an equation that describes how the percent signal received is related to the angle between the planes?

Answer: $y = 1.111x$

This is a great opportunity to show students how to perform simple statistical analysis on graphing calculators or with Excel. A linear regression can be performed with this data to produce the equation and graph of a line.



Activity I: How Does a Radio Work?

Part B - Probability and Reception

This section requires use of the chart from PartA.

For questions 1 to 6, the receiver can only be turned within the ranges for a, b, and c. (Hint: To make things easier, do 1 – 5 under the parameters for “a” first, then move on to “b” and “c”.)

- a. You may assume 0 = no signal and 90 = full signal.
- b. You may assume 0 = no signal, 90 = full signal, and 180 = no signal.
- c. You may assume 0 and 180 = no signal, and 90 and 270 = full signal.

1. What is the probability that you will receive a signal of 75% or greater if you randomly pick an angle to turn the receiver at?

- a) about 25% b) same as a c) same as a

2. If you randomly turn the receiver once every minute, on average, how long will it take for you to receive 100% signal?

90 minutes

3. If a person keeps turning the receiver once every minute and does not duplicate negative (<75%) signal angles, how long until the person will receive 100% signal?

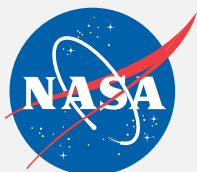
45.5 minutes

4. Luckily, once people see a signal, they don't keep turning the receiver randomly. Instead, once the person gets a positive signal, that person keeps turning it in a more positive direction. If a person keeps moving the receiver once every minute and keeps turning the receiver in the positive direction, and the person can only move it 5 degrees at a time, how long until (s)he reaches 100%?

18 minutes

5. Which situation is most realistic for a person adjusting their TV? Explain.

Answer: Students will likely pick #3 or #4, where turning is dependent on a response to signal, rather than being entirely random.



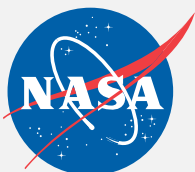
Activity I: How Does a Radio Work?

Part C - Understanding the Relationship Between Frequency, Capacitance, and Inductance

As soon as the electric field comes in contact with the receiving wire, the wire is “activated” and an electrical impulse is sent across the receiving wire through other wires, finally reaching the radio, where it results in sound. The frequency (f_o) of the sound received depends on the variable capacitor (C) and the inductor (L), as described previously.

The exact relationship between these variables is defined by the following equation:

$$f_o = \frac{1}{2\pi((LC)^{1/2})}$$



Activity I: How Does a Radio Work?

1. Fill in the following chart, showing that you understand the relationship between these variables.

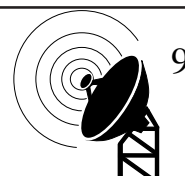
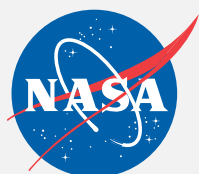
Note that frequency in the chart is in kilocycles, but frequency in the equation should be in cycles, which means students should multiply by 1000 to get f_0 in cycles.

Before doing this, you might lead students to rewrite the aforementioned equation to facilitate solving for L or C. Doing so may give a preview to the relationship to L and C, as they are largely interchangeable.

$$C = \frac{1}{(f_0 2\pi(L^{1/2}))^2}$$

Frequency, Capacitance, and Inductance

Frequency (kilocycles/second)	Capacitance (C)	Inductance (L)
30	2.50×10^{-4}	1.13×10^{-7}
30	1.00×10^{-4}	2.81×10^{-7}
30	5.63	5.00×10^{-12}
30	2.81	1.00×10^{-11}
3,000	5.00×10^{-4}	5.63×10^{-12}
3,000	1.00×10^{-4}	2.81×10^{-11}
3,000	1.13×10^{-6}	2.50×10^{-9}
3,000	9.38×10^{-7}	3.00×10^{-9}
30,000	15.00	1.88×10^{-18}
30,000	20.00	1.41×10^{-18}
30,000	1.13×10^{-18}	25.00
30,000	9.38×10^{-19}	30.00
300,000	2.00×10^{-14}	1.41×10^{-5}
300,000	4.00×10^{-14}	7.04×10^{-6}
300,000	4.69×10^{-16}	6.00×10^{-4}
300,000	3.52×10^{-16}	8.00×10^{-4}



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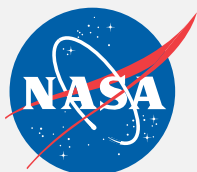
Discuss with students how to make conclusions about data by doing controlled comparisons, prior to answering the following questions.

- As the capacitor and inductor values get smaller, the frequency values get larger.
Cite example(s): comparison of 1st and 6th row
- For a single frequency, as the capacitor value gets smaller, the inductor value gets bigger.
Cite example(s): comparison of 1st and 2nd row
- As the product of inductor and capacitor values grows, the frequency gets smaller.
Cite example(s): comparison of 2nd and 6th row
- If the capacitor value is 10×10^{-12} and the indicator value is 30×10^{-4} , the frequency is about 920 kilocycles or kiloHertz.
- If the inductor value is 10×10^{-12} and the capacitor value is 30×10^{-4} , the frequency is same as #5.
- What do you notice about inductor, capacitor, and frequency settings in #5 and #6? Capacitance and Inductance are interchangeable.

Some common/familiar frequencies are listed in the following table.

Common Radio Frequencies

Frequency (kc or kHz)	Common Use
535– 1,605	AM radio
88,000– 108,000	FM radio
54,000– 88,000	TV channels 2– 6
174,000– 216,000	TV channels 7– 13
3,000– 30,000	Ship-to-ship and ship-to-shore communication
30,000– 300,000	Very High Frequency (VHF): aircraft-to-aircraft and air navigation, some TV
300,000– 3,000,000	Ultra High Frequency (UHF): some TV transmission



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8. Based on what you see in the Common Radio Frequencies chart, why do you think airline attendants ask passengers to turn off electronic devices including phones, radios, TVs, and computers during take-off and landing?

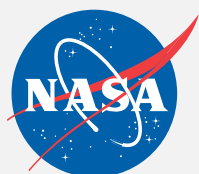
Answer: Airlines may communicate at frequencies also used by televisions and radios. As phones and computers use much of the same technology, they may also use these frequencies. Such use may interfere with airplane communication.

9. If you wanted to set your radio's receiving end for the following uses, to what would you set the different components?

Use a radio to show that radio station numbers are associated to their frequencies! Most radios have kHz for AM stations and MHz for FM stations. Turning the tuning knob changes the inductor and/or capacitor for the radio; this is why a new station is "heard."

Radio Communications

Use	Frequency	Inductance (L)	Capacitance (C)
Popular AM News Radio Station	740 kHz	5.00×10^{-4}	9.25×10^{-11}
Popular FM News Radio Station	104.9 MHz or 104900 kHz	1.00×10^{-7}	2.30×10^{-11}
Your favorite radio station	535 to 108,000 kHz	3.50×10^{-5}	2.53×10^{-9} to 6.21×10^{-14}
Communication between a lighthouse and a fishing boat	25,000 kc	7.00×10^{-6}	5.79×10^{-12}
TWA #218 to San Francisco International Airport	270,000 kc	4.00×10^{-7}	8.96×10^{-13}



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Part D - Understanding Components of Waves

Frequencies also have to do with some physical characteristics of the waves. Based on what you have done above, you now know how to alter radios so they can receive certain frequencies. But how can the provider of the sound change their frequencies? To understand this, we need to know something about waves.

Waves have 5 main characteristics:

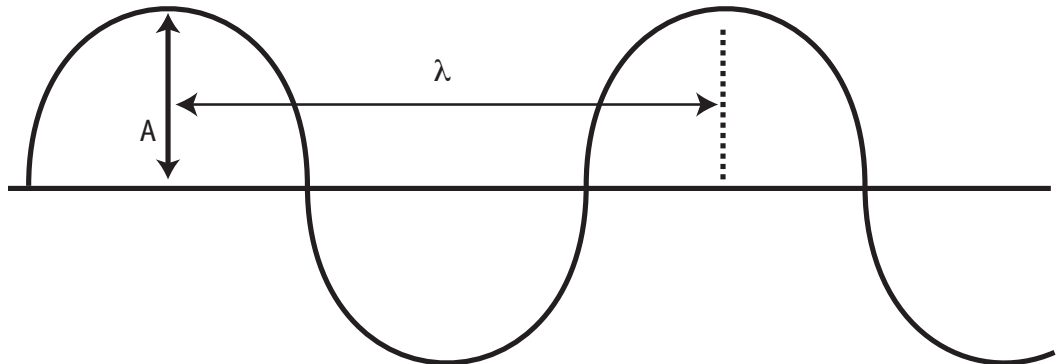
Frequency (f_0) = # of cycles / unit time

Amplitude (A) = 1/2 distance between maximal and minimum values obtained by function.

Wavelength (λ) = horizontal length of one cycle of wave

Period (T) = time required for one cycle of wave to move past a given point / observer. Reciprocal of frequency.

Phase Shift = least positive or greatest negative horizontal translation that maps $\sin x$ or $\cos x$ onto the given wave; delay in time when sine or cosine happens, as if it were to start at origin.



In math, there are several wave types easily defined by trigonometric equations. For instance, sine and cosine make beautiful waves, commonly referred to in science, technology, and math fields. The sine wave is also known as a wave representing pure tone.

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Use a graphing calculator to see which of the 5 characteristics of waves are controlled by the following variables. Do this by entering the following equations one-by-one and writing what you observe about them with respect to the first equation, $y = \sin x$.

1. $y = \sin x$

Sketch:

2. $y = \sin 2x$

Observation: half wavelength; otherwise same as $\sin x$

Sketch:

3. $y = 2\sin x$

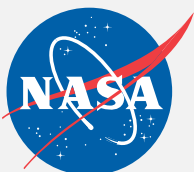
Observation: twice amplitude; otherwise same as $\sin x$

Sketch:

4. $y = \sin x + 2$

Observation: shift up two units; otherwise same as $\sin x$

Sketch:



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5. $y = \sin(x + 2)$ Observation: phase shift left two units; otherwise same
Sketch:

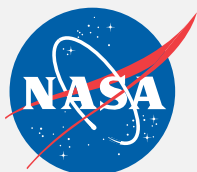
6. $y = (1/2)\sin x$ Observation: half amplitude; otherwise same as $\sin x$
Sketch:

7. $y = \sin(x/2)$ Observation: twice wavelength; otherwise same as $\sin x$
Sketch:

8. $y = 2/(\sin x)$ Observation: wavelengths are same, but all other features typical of a sine wave are lost
Sketch:

9. In the sine equations #2 to #5, replace all twos with negative twos, and observe the changes. Mark the new graphs on your previous sketches, in a different color.

Students should notice in #2 and #3 that the initial direction of the curve changes. In #4, vertical shift is down instead of up. In #5, phase shift is right instead of left.



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10. For the equation $y = a \sin(bx + c) + d$,
a changes Amplitude, b changes Wavelength (by reciprocal),
c changes Phase Shift (by opposite), and d changes Vertical Shift.

11. How was use of fractions to investigate especially helpful to clarify the influence of b on the equation?

Answer: Use of $1/2$ clearly showed the increase in wavelength by its reciprocal(2).

12. What happens when a or b are made negative?

Answer: Sine curve changes direction.

13. What did negative c and d show you?

Answer: When c is negative, phase shift occurs to the right. When d is negative, there is a downward vertical shift.



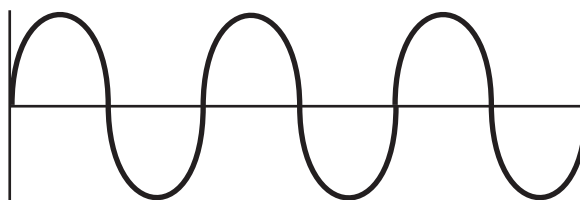
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The following waves are produced by a variety of instruments, all creating the A note. Approximate the equations for each frequency, using what you now know about sine waves and your graphing calculator.

Students may need assistance in finding the underlying wave forms. Encourage them to look for symmetry.

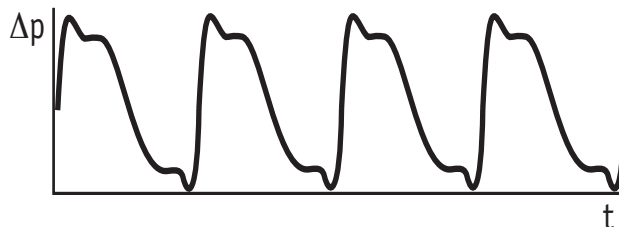
14. Pure Tone (Tuning Fork) wave form:

$$y = \sin x$$



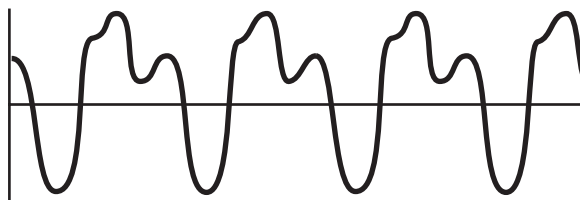
15. Piano wave form:

$$y = \sin(4/3)x$$



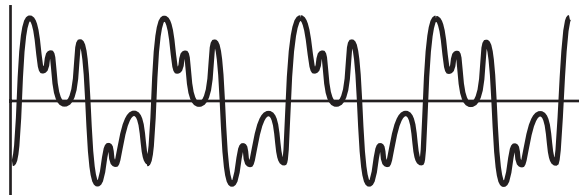
16. Flute wave form:

$$y = \sin((32/25)x - 4/25) \text{ OR } \sin((16/13)x - 2/13)$$



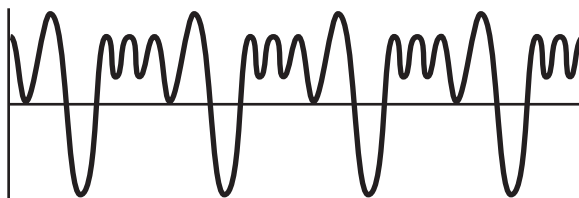
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17. Clarinet wave form:



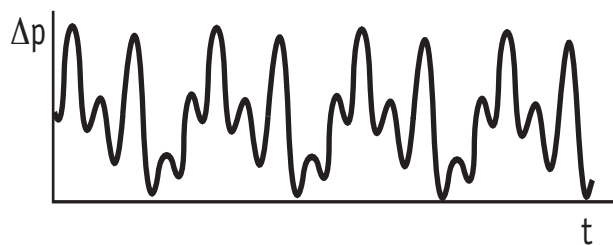
$$y = \sin\left(\frac{16}{11}x - \frac{1}{22}\right) \text{ OR } \sin\left(\frac{32}{23}x - \frac{1}{23}\right)$$

18. Trumpet wave form:



$$y = \sin\left(\frac{8}{3}x - \frac{1}{4}\right) \text{ OR } \sin\left(\frac{8}{9}x - \frac{1}{4}\right)$$

19. Violin wave form:



$$y = \sin\left(\frac{16}{5}x - \frac{1}{10}\right)$$

Activity I: How Does Radio Work?

Because only the pure tone shows a “clean” wave, this is a good opportunity to talk about what makes the instruments’ waves less “clean,” harmonics, etc. How come tuning forks never go out of tune, but we always have to tune our instruments?

There are 2 web sites with sound and animation of different tones and their respective graphs.

At www.haskins.yale.edu/Haskins/MISC/SWS/SWS.html researchers are trying to generate synthetic speech complete with inflections by simply using math. They are essentially working in the direction opposite of what students did in questions 12 through 17 of this section. The scientists start with the equation and then generate the sound. It is eerie how similar, yet clearly synthetic, the computer-generated sentences are to human speech.

At www.doctrionics.co.uk/scope.htm researchers not only provide nice background on waves, but also have graphical examples of waves with sound. Students can use this source to practice writing more sin equations. The web page also includes nice quizzes (with keys) regarding wave principles.

